



Worksheet 5 Problem recognition **Answers**

Task 1

1. Write an algorithm to allow the user to enter an integer number for the number of paper bags, and a second integer (which must be greater than the first) for the number of sweets. The program then tells the user whether it is possible to put an odd number of sweets in each bag.

You cannot put an odd number of sweets in each of an even number of bags.

If you add 2 odd numbers you get an even number. the same is true if you add $2n$ odd numbers, whatever the value of n .

Similarly, you cannot put an even number ($2n$) sweets in an odd number of bags so that each bag contains an odd number.

There are many ways of putting an odd number of sweets in an odd number of bags, or an even number of sweets in an even number of bags, so that each bag contains an odd number.

The simplest is to put one sweet in each bag and all the remaining sweets in the first bag. Some students may come up with algorithms to distribute the sweets more evenly - but is that a "better" solution?

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b = INPUT "number of bags"
s = INPUT "number of sweets"
IF b is even AND s is odd THEN
    OUTPUT "Not possible"
ELSE IF b is odd AND s is even THEN
    OUTPUT "Not possible"
ELSE
    put one sweet in each bag
    put remainder in first bag
ENDIF
```

Task 2

2. There are many way of solving a problem, including:

- simulation
- enumeration - list all cases
- trial and error
- theoretical approach
- creative solution

- a. Which of the methods listed above could you use to find the cube root of 729?

- (i) Write down the answer (what is the cube root of 729)?



Enumeration, combined with trial and error, is the simplest! Take a guess; it must be an odd number, and we know 10^3 is 1000 so it must be less than 10. Try 7. That gives 343. Try 9 – Bingo!

- (ii) Write down the steps you took to find the answer

See part i.

- b. What method(s) could you use to estimate the probability of throwing a double six with two dice?

You could use a theoretical approach, if you are familiar with probability theory. Otherwise, you could use the simulation method and simulate throwing a dice say 1000 times, 5000 times etc., until you are confident of the answer.

- c. Add up all the numbers between 1 and 50. What method of solution did you use?

You could use enumeration but a better way is the theoretical approach – there is a formula for this. To work out the formula:

$$\begin{array}{r} 1 + 2 + 3 + \dots + 50 \\ 50 + 49 + 48 + \dots + 1 \\ \hline = 51 + 51 + 51 + \dots + 51 = (50 * 51) / 2 \end{array}$$

Sum of numbers $1..n = n(n+1)$

$$\frac{\quad}{2}$$

- d. On a computer network, if two devices using the same line try to transmit at exactly the same time, a “collision” occurs. The network detects the collision and both transmissions are discarded. Can you think of a solution to this problem? Which of the problem-solving methods is applicable?

This is a real problem for which a creative solution was found. Each station waits a random amount of time before retransmitting. Chances of another collision are extremely small.

- e. Scientists working at Bletchley Park on the Enigma code during WWII eventually managed to decode the messages sent by the Germans, even though the “key” was changed daily. What problem solving techniques do you think would be effective in cracking what was supposed to be an “uncrackable” code?

They tried the “brute force” method, i.e. enumeration, trying all possibilities but they could not crack the code in a day before the key was changed again. The insight was a discovery that several messages ended in ‘Heil Hitler’, or began in a particular way, and this gave them enough letters to be able to crack the rest.



- f. Environmental scientists want to study the effects on the rabbit population in a particular area if a cull of foxes is carried out. What problem-solving method could be applied?

Simulation is the classic technique here.



Task 3

3. This is an example of the ***Decrease and Conquer*** strategy.

A group of 10 Venture Scouts are stranded on a small island, a short distance from the mainland. Two small boys are playing on the shore in a very small rowing boat, which is only big enough to hold either the two boys or one Venture Scout.

How can all the Venture Scouts reach the mainland and leave the boys and their boat together on the island?

How many trips does the boat make from one shore to the other?

What is the answer in the general case of n Venture Scouts?

The Venture Scouts persuade the two boys to cross to the far shore, and one of them returns with the boat. Then a scout takes the boat to the other side and the small boy returns. These four trips reduce the size of the problem by one. Repeat this procedure 9 times, making 40 crossings in all.

In the general case of n scouts, the answer is $4n$.

Task 4

4. You are lost in a jungle, walking along a narrow path. You come to a T-junction, and you are aware that one way leads out of the jungle to safety, the other to a snake infested area and almost certain death. There are two tribesmen at the T junction, and you have been informed that one of these men will always answer a question truthfully, the other will always lie. What question will you ask?

Some creative thinking required here! Ask either of them:

"If I were to ask the other tribesman which way leads to safety, what would he say?"

Then take the other path.